

INVITED LECTURES,
RECENT ADVANCES IN CLASSICAL ALGEBRAIC GEOMETRY,
27 JUNE 2022 - 2 JULY 2022

Speaker: Olivier Benoist (École Normale Supérieure Paris)

Title: Smooth subvarieties of Jacobians.

Abstract: I will present new examples of algebraic cohomology classes on smooth projective complex varieties that are not integral linear combinations of classes of smooth subvarieties. Some of these examples have dimension 6, the lowest possible. More precisely, I will consider the case of minimal cohomology classes on Jacobians of very general curves. This is joint work with Olivier Debarre.

Speaker: Jérémy Blanc (Universität Basel)

Title: From the birational geometry of pointless surfaces to the Cremona group.

Abstract: In this talk, I will describe the birational geometry of non-trivial Severi Brauer surfaces, proving in particular that it is not generated by elements of finite order and apply it to the Cremona groups over the complex numbers. Universality of the quotients and the torsion of the abelianisation of the groups of birational transformations will be of particular interest. Joint work with Julia Schneider and Egor Yasinsky.

Speaker: Christian Böhning (University of Warwick)

Title: Skew matrices of linear forms, matrix factorisations and intermediate Jacobians of cubic threefolds

Abstract: I will report on some ongoing joint work with Hans-Christian von Bothmer (Hamburg) and Lukas Buhr (Mainz).

Results due to Druel and Beauville show that the blowup of the intermediate Jacobian of a smooth cubic threefold X in the Fano surface of lines can be identified with a moduli space of semistable sheaves of Chern classes $c_1 = 0$, $c_2 = 2$, $c_3 = 0$ on X . We identify this space with a space of matrix factorisations. This has the advantage that this description naturally generalises to singular and even reducible cubic threefolds. In this way, given a degeneration of X to a reducible cubic threefold X_0 , we obtain an associated degeneration of the above moduli spaces of semistable sheaves.

Speaker: Ana-Maria Castravet (Université Paris-Saclay)

Title: Blown up toric surfaces with non-polyhedral effective cone

Abstract: In recent joint work with Antonio Laface, Jenia Tevelev and Luca Ugaglia, we construct, in every characteristic, examples of projective toric surfaces whose blow-up at a general point has a non-polyhedral pseudo-effective cone. As a consequence, we prove that the pseudo-effective cone of the Grothendieck-Knudsen

moduli space of stable rational curves with n markings is not polyhedral for $n \geq 10$. In this talk I will discuss an interesting class of arithmetic threefolds that we call arithmetic elliptic pairs of infinite order, which allows us to construct blow-ups of toric surfaces - defined over the rational numbers - which have a non-polyhedral effective cone in characteristic zero and in characteristic p for an infinite set of primes p of positive density.

Margherita Lelli-Chiesa (Scuola Normale Superiore di Pisa)

Title: Irreducibility of Severi varieties on K3 surfaces

Abstract: Let (S, L) be a general K3 surface of genus g . I will prove that the closure in $|L|$ of the Severi variety parametrizing curves in $|L|$ of geometric genus h is connected for $h \geq 1$ and irreducible for $h \geq 4$, as predicted by a well known conjecture. This is joint work with Andrea Bruno.

Speaker: Kento Fujita (Osaka University)

Title: The Calabi problem for Fano threefolds.

Abstract There are 105 irreducible families of smooth Fano threefolds, which have been classified by Iskovskikh, Mori and Mukai. For each family, we determine whether its general member admits a Kaehler-Einstein metric or not. This is a joint work with Carolina Araujo, Ana-Maria Castravet, Ivan Cheltsov, Anne-Sophie Kaloghiros, Jesus Martinez-Garcia, Constantin Shramov, Hendrik Suess and Nivedita Viswanathan.

Speaker: Joachim Jelisiejew (Uniwersytet Warszawski)

Title: Hilbert schemes of points: bad spaces with nice topology.

Abstract: Two recent results on Hilbert schemes of points are in contrast: first, they are shown to have very pathological singularities. Second, their topology, at least in the limit, is very simple. In the talk I will report on these results, mostly the topological part, speculating on more structure behind these results. The motivation for all this comes from motivic homotopy theory.

Speaker: Michael Kemeny (University of Wisconsin - Madison)

Title: The rank of syzygies.

Abstract: I will explain a notion of rank for the relations amongst the equations of a projective variety, generalizing the classical notion of rank of a quadric. I will then turn to explain a result telling us that, for a general canonical curve, all syzygies are linear combinations of syzygies of minimal rank. This is a sweeping generalization of old results of Andreotti-Mayer, Harris-Arbarello and Green, which tell us that canonical curves are defined by quadrics of rank four. As a special case, this perspective gives us a new, and simpler, proof of Green's conjecture for general curves.

Speaker: Eric Larson (Stanford University)

Title: Interpolation for Brill-Noether Curves.

Abstract: In this talk, we determine when there is a Brill–Noether curve of given degree and given genus that passes through a given number of general points in any projective space..

Speaker: Emanuele Macrì (Université Paris-Saclay)

Title: Lagrangian fibrations on hyper-Kähler fourfolds.

Abstract: I will present joint work with Olivier Debarre, Daniel Huybrechts and Claire Voisin on the SYZ hyper-Kähler conjecture for fourfolds under certain topological assumptions. As application, this proves a conjecture by O’Grady that a hyper-Kähler fourfold whose cohomology ring is isomorphic to the one of the Hilbert square of a K3 surface is a deformation of a Hilbert square.

Speaker: Yuji Odaka (Kyoto University)

Title: Type II degeneration of K3 surfaces and type III degeneration of elliptic K3 surfaces via Piecewise-linear functions on the interval

Abstract: We relate type II degeneration of K3 surfaces and type III degeneration of elliptic K3 surfaces. Then, we associate explicit PL functions to either of them in a compatible way. It can be regarded as a new invariant for the degenerations, which refines the classical root lattices for them in the literature (e.g. Friedman in the 80s analysed degree 2 K3s case). We encountered these in the context of limits of hyperKähler metrics, and the relation is one of our results. On the other hand, main methods for the proof are to use (various) moduli compactifications. Main reference is arXiv:2010.00416, published from Nagoya Math J, after a general context of O-Oshima Memoir MSJ vol40.

Speaker: John Ottem (Universitetet i Oslo)

Title: Two Coniveau Filtrations

Abstract: A cohomology class of a smooth complex variety of dimension n is said to be of “coniveau” at least c if it vanishes on the complement of a closed subvariety of codimension at least c , and of “strong coniveau” at least c if it comes by proper pushforward from the cohomology of a smooth variety of dimension at most $n-c$. These notions give rise to two filtrations on the cohomology groups of a variety, which are known to coincide in many cases (for instance, they agree on the rational cohomology of any smooth projective variety). However, we show that they differ in general, both for integral classes on smooth projective varieties and for rational classes on smooth open varieties. The difference between the two filtrations also give rise to new birational invariants. The talk will give an introduction to the circle of ideas surrounding these two filtrations, with lots of explicit examples and constructions. This is joint work with Olivier Benoist.

Speaker: Zolt Patakfalvi (École Polytechnique Fédérale de Lausanne)

Title: Varieties with ample Frobenius-trace kernel.

Abstract: Let X be a projective variety over an algebraically closed field k . Let us recall first the following :

- (1) We know that X is smooth if and only if the cotangent bundle of X is locally free.
- (2) The global version of this fundamental fact is Mori's celebrated theorem stating that X is isomorphic to a projective space if and only if the cotangent bundle is anti-ample and locally free.
- (3) If the characteristic of k is $p > 0$, then there is a variant of the local condition (1) saying that X is smooth if and only if certain sheaf associated to the absolute Frobenius morphism is locally free. More specifically, this sheaf is the cokernel of the homomorphism on the structure sheaves associated to the absolute homomorphism. This is a difficult commutative algebra theorem shown by Kunz.

Given the above theorems, it is natural to ask if there is a global version of Kunz's theorem in the spirit of Mori's characterization of projective spaces. I will present both negative and positive results in this direction. This is a joint work with Javier Carvajal-Rojas.

Speaker: Yuri Prokhorov (Steklov Mathematical Institute)

Title: Singular Del Pezzo varieties.

Abstract: A del Pezzo variety X is a Fano variety whose anticanonical class has the form

$$-K_X = (n-1)A,$$

where A is an ample line bundle and n is the dimension of X . This is a higher dimensional analog the notion of del Pezzo surfaces. I am going to discuss biregular and birational classifications of del Pezzo varieties admitting terminal singularities.

The talk is based on a joint work with Alexander Kuznetsov (in preparation).

Speaker: Claudiu Raicu (University of Notre Dame)

Title: Koszul modules.

Abstract: The goal of my talk is to introduce the basic theory of Koszul modules, and discuss a number of applications and open problems related to them. Koszul modules were originally introduced by Papadima and Suciu as topological invariants of groups, but more recently a more algebraic theory has emerged, which has resulted in new proofs of the generic Green conjecture, the canonical ribbon conjecture, and a number of interesting consequences to the study of algebraic varieties and that of finitely generated groups. Joint work with M. Aprodu, G. Farkas, S. Papadima, S. Sam, A. Suciu, J. Weyman.

Speaker: Giulia Saccà (Columbia University New York)

Title: Moduli spaces as Irreducible Symplectic Varieties.

Abstract: Recent developments by Druel, Greb-Guenancia-Kebekus, Horing-Peternell have led to the formulation of a decomposition theorem for singular (klt) projective varieties with numerical trivial canonical class. Irreducible symplectic varieties are one of the building blocks provided by this theorem, and the singular analogue of irreducible hyper-Kähler manifolds. In this talk I will show that moduli

spaces of Bridgeland stable objects on the Kuznetsov component of a cubic fourfold with respect to a generic stability condition are always projective irreducible symplectic varieties. This builds on the recent work of Bayer-Lahoz-Macri-Neuer-Perry-Stellari, which, ending a long series of results by several authors, proved the analogue statement in the smooth case.

Speaker: Stefan Schreieder (Leibniz Universität Hannover)

Title: A moving lemma for cohomology with support.

Abstract: We explain that Chow's moving lemma for algebraic cycles on smooth quasi-projective varieties has an analogue for pairs of a cycle and a homology class supported on the given cycle. This moving lemma generalizes in our setting the effacement theorem, resp. the Gersten conjecture, proven by Bloch—Ogus and Gabber. Several applications will be discussed.

Speaker: Susanna Zimmermann (Université d'Angers)

Title: Involutions of the real plane.

Abstract: What are the finite groups acting non-trivially by birational transformations on the plane? The answer to this question was pursued by many, starting with Bertini. The cumulative work of many mathematicians over many decades contributed to the classification we have today, given by Dolgachev-Iskovskih and Blanc, over the complex numbers. The particular classification of birational involutions of the complex plane was achieved by Bayle-Beauville - they are conjugate to linear maps, Jonquière maps, Geiser involutions or Bertini involutions. Moreover, the conjugacy classes of the latter three are in one-to-one correspondence with the isomorphism classes of their fixed curves.

This correspondence does not hold over the real numbers. In fact, there are many non-conjugate birational involutions of the real plane that have real isomorphic fixed curves. I will motivate this and give the classification of the birational involutions of the real plane with geometrically irrational fixed curve. This is joint work with I. Cheltsov, F. Mangolte and E. Yasinsky.

Speaker: Jakub Witaszek (University of Michigan Ann Arbor)

Title: Quasi-F-splittings.

Abstract: What allowed for many developments in algebraic geometry and commutative algebra was a discovery of the notion of a Frobenius splitting, which, briefly speaking, detects how pathological positive characteristic Fano and Calabi-Yau varieties can be. Recently, Yobuko introduced a more general concept, a quasi-F-splitting, which captures much more refined arithmetic invariants. In my talk, I will discuss on-going projects in which we develop the theory of quasi-F-splittings in the context of birational geometry and derive applications, for example, to liftability of singularities. This is joint work with Tatsuro Kawakami, Hiromu Tanaka, Teppei Takamatsu, Fuetaro Yobuko, and Shou Yoshikawa.

Speaker: Ziquan Zhuang (Massachusetts Institute of Technology)

Title: Finite generation and K-stability

Abstract: Originally motivated by the Calabi problem on the existence of Kähler-Einstein metrics on Fano varieties, K-stability has evolved into an active algebraic subject that's intimately related to birational geometry. In this talk, I will survey some recent progress in the local (concerning klt singularities) and global (concerning Fano varieties) aspects of K-stability, focusing on their connections to the higher rank finite generation conjecture in birational geometry.